Frequency Control and Its Effect on the Dynamic Response of Flexible Structures

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The effect of structural optimization on optimal control design is studied in this paper. Structural optimization was treated as a problem of mass minimization with constraint on the open-loop frequency. The quadratic performance index, involving the state and control variables, was used in the design of the control system. A control system with only full-state feedback was considered. A procedure for generating the state and control weighting matrices by structural dynamics programs was outlined. By introducing simple scaling parameters, the weighting matrices were used effectively to achieve the desired control objectives. A number of case studies using a simple truss structure were made, in which vibration suppression with only initial disturbances was considered. The conclusion was that modification of the structural parameters (stiffness and structural mass) did not significantly alter the control design in this study.

Introduction

VIBRATION control is an important consideration in the design of dynamic systems on the ground, in the air, and in space. The disturbances in ground and air vehicles are primarily caused by rough road (runway) profiles and airflow, such as gusts and powerplants. Similarly, in large space structures the disturbances are the result of slewing/pointing maneuvers, thermal transients, and mechanical machinery such as coolers, generators, etc. Control of the dynamic response is essential for maintaining the ride quality and performance requirements, as well as for the safety of the structure.

The response of a structure is basically governed by three sets of parameters. The mass, damping, and stiffness represent the structural parameters. The second set of parameters is due to the sources of external disturbances. These are generally external to the system and are considered as fixed inputs; thus their alteration is not within the realm of the structures/controls designer. The third set represents the control system, assuming that the structure is actively controlled. Control of the dynamic response by modification of the structural parameters alone is considered to be passive. Passive control is most appealing from both the reliability and maintainability points of view, if it can be achieved at all economically. Basically, the stiffness and mass modifications result in frequency and mode changes, while the damping affects the dissipation energy of the system. The damping can be significantly altered by either viscoelastic coatings (or constrained layer damping) or the provision of discrete dashpot mechanisms.

The objective of vibration control is to design the structure and its controls either to eliminate vibration completely or to reduce the mean square response of the system to a desired level within a reasonable span of time. In addition, it is important that this objective be achieved in some optimal way. For a structural designer, the optimal design represents an adjustment of structural mass while improving the dynamic characteristics (such as changes in frequencies and mode shapes) to reduce the dynamic response. For a control designer, on the other hand, optimization represents sizing of the controllers in such a way that a specified performance index is minimized. If the locations and the number of actuators and sensors are predetermined and fixed, then the individual actuator inputs are the variables in the control optimization problem. Otherwise, both the number and the locations of the actuators can also be variables in the optimal control design.

The interaction between the structures and controls designers has been very minimal in the past. The structural designer develops his designs based on strength and stiffness requirements derived from the peak maneuver loads expected during the operation of the flight vehicle. His primary concern is to design lightweight (minimum mass) structures that satisfy the strength, stiffness, and other performance requirements. In general, the designer of active controls has little input in the evolution of the basic structural design. Similarly, the structural analyst's participation in the control design is, at best, limited to providing information about the frequencies and mode shapes of the primary structure. This practice of compartmentalizing the design process is promoted by the attitude that optimal controls can be designed for any structure and vice versa. To a great extent, this may be true because many successful systems were designed in the past based on this philosophy. However, there has been strong indication in recent years that significant performance as well as cost improvements can be realized by optimizing the structure and the controls together. There is general agreement[1] that future research in large space structures dynamics and controls should develop algorithms to promote such an interdisciplinary approach to design. Recent advances in digital computers and computational algorithms provide ample opportunity for such development. Reference 2 contains a number of papers supporting this theme and also some ideas about how to formulate the combined optimization problem. However, there is also a realization that, for a number of reasons, this combined optimization problem is not easily tractable. For example, in structural optimization, treatment of time response constraints can be quite involved and the solution is, in general, not unique. Multiple minimums with the same or different merits can exist. On the other hand, the linear optimal control problem is

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naturally a time response problem and when the solution exists it is unique. The issues of how the relative minimums in structural optimization affect the control optimization problem are not clear and must be explored.

The object of this paper is to study the effects of modification of structural parameters on the dynamic response of flexible structures with and without active controls. The resulting automated analysis procedure is expected to be a precursor to the development of a sensitivity analysis in which the issues of both optimal structure and controls can be addressed. The problem will be treated as having dual objective functions. The optimization will be carried out independently, but the combined effect of the structural optimization and controls on the dynamic response will be studied. The structural optimization will be posed as a problem of the minimization of structural mass with the constraint on the fundamental frequency. In the optimal control design, a quadratic performance function involving the energy of the vibrating structure and the actuators will be minimized.

Equations of Motion and Control Objectives

The second-order differential equation that governs the forced vibration of a large space structure with active controls can be written as

$$m(\underline{d})\ddot{r}(t)+c(\underline{d})\dot{r}(t)+k(\underline{d})r(t)=F_1(u,t)+F_2(t) \tag{1}$$

The n-coupled differential equations represent the mathematical model of a discretized (finite element model) large space structure. The $n\times 1$ vector $r(t)$ represents the system response in the configuration space. The $n\times n$ mass m, damping c, and stiffness k matrices are functions of the structural variables $\underline{d}$. The number of structural variables can be as few as one or as many as a multiple of the number of finite elements in the structure. In addition, the mass matrix generally consists of structural and nonstructural components. Only the structural mass will be a function of the structural variables. In a finite element formulation, the mass and stiffness matrices are symmetric and positive definite or at least semidefinite. The damping matrix will be assumed proportional to the stiffness and the mass matrices in this formulation

$$c(\underline{d})=\alpha k(\underline{d})+\beta m(\underline{d}) \tag{2}$$

where α and β are proportionality constants.

The right side of Eq. (1) contains two parts: $F_1(u,t)$ represents the control input and the function $F_2(t)$ represents the external disturbances that initiate or continue the vibration of the system. In this investigation, only the vibration initiation will be considered (no forced motion, $F_2(t)=0$). In such a case, the governing differential equation can be written as

$$m(\underline{d})\ddot{r}(t)+c(\underline{d})\dot{r}(t)+k(\underline{d})r(t)=bu(t) \tag{3}$$

It is assumed that the control system consists of a set of discrete actuators. The $m\times 1$ vector $u(t)$ represents the input of the actuators, while b is an $n\times m$ matrix that identifies the position and the relationship between the controllers and the actuators.

The state space representation of the governing differential equations for the open-loop system is written as[3-8]

$$\dot{X}=AX+Bu \tag{4}$$

where the $2n\times 1$ vectors $\dot{X}$ and X are given by

$$\dot{X}=\begin{bmatrix}\ddot{r}\\ \hline \dot{r}\end{bmatrix}_{2n\times 1} \qquad X=\begin{bmatrix}\dot{r}\\ \hline r\end{bmatrix}_{2n\times 1} \tag{5}$$

The open-loop plant matrix A and the control matrix B are given by

$$A=\left[\begin{array}{c|c}-m^{-1}c & -m^{-1}k\\ \hline I & 0\end{array}\right]_{2n\times 2n} \qquad B=\begin{bmatrix}m^{-1}b\\ \hline 0\end{bmatrix}_{2n\times m} \tag{6}$$

The governing differential equations as given in Eqs. (4-6) represent the full system formulation without modal reduction. The usual procedure to reduce the dimensionality of the system is by modal reduction, which is obtained by substituting

$$r(t)=\Phi\eta(t) \tag{7}$$

in Eq. (3). The $p\times 1$ vector $\eta(t)$ represents the normal coordinates. The matrix Φ is an $n\times p$ modal matrix, which is a solution of the eigenvalue problem

$$\omega^2 mr=kr \tag{8}$$

The number of modes ($p\leq n$) required to represent the dynamic response depends on the type of disturbances and the number and locations of the actuators and sensors. ω^2 represents the eigenvalues of the system.

In the case of modal reduction the state vectors $\dot{X}$ and X in Eq. (4) are given by

$$\dot{X}=\begin{bmatrix}\ddot{\eta}\\ \hline \dot{\eta}\end{bmatrix}_{2p\times 1} \qquad X=\begin{bmatrix}\dot{\eta}\\ \hline \eta\end{bmatrix}_{2p\times 1} \tag{9}$$

Now the plant and the control matrices of the open-loop system are given by

$$A=\left[\begin{array}{c|c}[-2\zeta_i\omega_i] & [-\omega_i^2]\\ \hline [I] & [0]\end{array}\right]_{2p\times 2p} \qquad B=\begin{bmatrix}\Phi^t b\\ \hline 0\end{bmatrix}_{2p\times m} \tag{10}$$

The submatrices in A are all diagonal matrices. The damping ratio ζ_i is given by

$$\zeta_i=\frac{\alpha}{2/\omega_i}+\frac{\beta}{2\omega_i} \tag{11}$$

The damping ratio $\zeta_i<1$, which represents the underdamped case, is of interest to this discussion.

Returning to the open-loop plant of the full system formulation

$$\dot{X}=AX+Bu \tag{12}$$

the output of the system can be represented by

$$y_{s\times 1}=C_{s\times 2n}X_{2n\times 1} \tag{13}$$

Equation (13) includes both the velocity and the displacement sensors. $s=2n$ represents the case where there are enough sensors to measure the entire state. For most practical systems, this would be an unrealistic assumption. $s<2n$ represents a more realistic case. Basically, the C matrix will contain only zeros and ones. The ones correspond to where the sensors are located.

The output as given by Eq. (13) represents the ideal state, while the actual observer state and the corresponding output will be represented by

$$\dot{\bar{X}}=A\bar{X}+Bu+L(y-\bar{y}) \tag{14}$$

where $\bar{y}$ is given by

$$\bar{y}=C\bar{X} \tag{15}$$

and L is a $2n \times s$ observer matrix. The elements of the observer matrix can be determined by an eigenvalue and eigenvector assignment.[3] However, for this investigation the full state feedback is used.

As stated earlier, the control variables are given by an $m \times 1$ vector u which represents the inputs from the m actuators. To determine the optimal state feedback control law, modern control theory minimizes a quadratic performance index (PI), which is a function of the state and control vectors $X(t)$ and $u(t)$, and is given by

$$\text{PI} = \int_0^\infty (X^t Q X + u^t R u) \mathrm{d}t \tag{16}$$

where Q and R are weighting matrices. Their selection can be somewhat arbitrary except that they must satisfy the requirements of positive definiteness. Matrix R must be positive definite, while Q must at least be positive semidefinite. By adjusting these weighting matrices, the control objectives such as amplitude of dynamic response, settling time (t_s), etc., can be altered. In addition, the weighting matrices can also be used as a means of imposing amplitude constraints on the control vector $u(t)$. A brief discussion of how the matrices Q and R are selected for this investigation is given in the next section.

The result of the minimization of the quadratic performance index (while satisfying the state equation) is the optimal state feedback control law

$$u^*(t) = -GX^*(t) \tag{17}$$

where $u^*(t)$ is the optimal control input vector and $X^*(t)$ the corresponding state. The feedback matrix (control gain matrix) G is given by

$$G = R^{-1} B^t P \tag{18}$$

where P is a symmetric positive definite matrix known as the Riccati matrix and is obtained by the solution of the algebraic Riccati equation

$$A^t P - PBR^{-1} B^t P + PA + Q = 0 \tag{19}$$

Substitution of Eq. (17) in Eq. (12) gives the governing equations for the optimal closed-loop system in the form

$$\dot{X}^* = A^* X^* \tag{20}$$

where the closed-loop plant matrix A^* is given by

$$A^* = A - BG \tag{21}$$

Now the standard procedure for the solution of the optimal control problem for a given plant, control system, and weighting matrices is as follows. The first step is to determine the controllability and observability of the system.

For this investigation, it was assumed that the sensors are located at the same place as the actuators and also that the true state is the observable state. The next step in the solution of the optimal control problem is to solve for the Riccati matrix P from Eq. (19). The third step is to determine the control gain matrix G by substituting the Riccati matrix P in Eq. (18). The fourth step is to determine the state transition matrix from the solution of Eq. (20). From the state transition matrix and the initial state, the optimal state can be determined. Then the control input vector $u^*(t)$ can be determined by Eq. (17). With this procedure, there is a facility to monitor the complete state, the control input, and the performance index PI. The weighting matrix R will be selected in such a way that the second term in the performance index contains all of the information about the power requirements of the individual actuators.

The Weighting Matrices Q and R

It is evident from the foregoing discussion that there are five important input matrices involved in the solution of the optimal control problem. The matrices A and B are unique to a given plant and control system. The matrix C is also unique for the desired output. The weighting matrices, on the other hand, are not unique and they can be selected to achieve certain desirable control objectives. However, the matrix R must be positive definite and the Q matrix must be at least positive semidefinite. The weighting matrices selected for this investigation are as follows:

$$Q = \left[\begin{array}{c|c} \theta_m^2 m & 0 \\ \hline 0 & \theta_k^2 k \end{array}\right]_{2n \times 2n} \tag{22}$$

and

$$R = \theta_R^t b^t k^{-1} b \theta_R$$

where the quantities θ_m and θ_k are the scaling parameters that can be adjusted to achieve the desired control objectives. θ_R is an $m \times m$ diagonal matrix that will be used for similar scaling purposes on the matrix R. Some of the possible parameters that can be adjusted are the amplitude of the dynamic response, the settling time t_s, etc. In addition, amplitude constraints on the control vector $u(t)$ can also be imposed.

In Eq. (22), k is assumed to be a nonsingular matrix. However, for a free-free structure k^{-1} does not exist, and the necessary modification is outlined in the sequel. The dimension of the singular matrix is $n \times n$ and its rank would be $n - \ell$, where ℓ represents the number of rigid-body degrees

Table 1 Characteristics of two structures

Element frequency no.	Member areas, cm^2 (in.^{2}) Original structure	Member areas, cm^2 (in.^{2}) Optimal structure	Frequencies, Hz Original structure	Frequencies, Hz Optimal structure
1	0.65 (0.1)	0.348 (0.054)	3.44	3.44
2	0.65 (0.1)	0.065 (0.010)	12.47	9.35
3	0.65 (0.1)	0.348 (0.054)	15.07	12.21
4	0.65 (0.1)	0.987 (0.153)	26.56	13.28
5	0.65 (0.1)	0.065 (0.010)	36.22	17.66
6	0.65 (0.1)	0.987 (0.153)	37.86	19.82
7	0.65 (0.1)	0.361 (0.056)	39.28	33.11
8	0.65 (0.1)	0.361 (0.056)	43.74	35.28
9	0.65 (0.1)	0.361 (0.056)		
10	0.65 (0.1)	0.361 (0.056)		
Structure weight, kg (lb)	2.21 (4.88)	1.56 (3.44)		

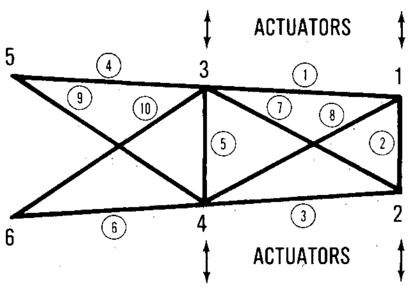

Fig. 1 Two-bay truss with four actuators.

of freedom of the system. In such a case, the weighting matrix R can be written symbolically as

$$R=\theta_R^t\,[\bar{b}^t \,\vdots\, 0]\begin{bmatrix}\bar{k} & 0\\ 0 & I\end{bmatrix}^{-1}\begin{bmatrix}\bar{b}\\ 0\end{bmatrix}\theta_R \tag{23}$$

where $\bar{k}$ is a nonsingular matrix of dimension $(n-\ell)\times(n-\ell)$. In other words, $\bar{k}$ is the original stiffness matrix with the rows and columns corresponding to the rigid-body degrees of freedom removed. The $\ell\times\ell$ identity matrix takes the place of the rigid-body degrees of freedom. It was assumed, for convenience, that the elastic and rigid-body degrees of freedom are neatly partitioned as shown. However, in an actual structure, these are generally interspersed and they can be handled routinely without much difficulty. In the foregoing discussion, it was tacitly assumed that a free-free structure will have two types of controllers, one set for controlling rigid-body modes and the other for elastic modes. The interest of the present paper is only to address controllers with authority over the elastic modes.

In a modal formulation, the R matrix remains the same as given by Eq. (22), but the Q matrix takes the following form:

$$Q=\begin{bmatrix}\theta_m^2 I & 0\\ 0 & \theta_k^2\omega_i^2\end{bmatrix} \tag{24}$$

Numerical Results and Discussion

The purpose of this numerical study is to highlight the effects of the changes in the parameters associated with the weighting matrices θ^s and the structural variables $\underline{d}$ on the dynamic response of an actively controlled structure. The dynamic response $X(t)$, the actuator input $u(t)$, the settling time t_s, and the performance index PI are a measure of the effectiveness of the control system. The object of vibration control is either to damp out the vibration completely or to bring it to a predetermined level in a finite settling time t_s. Another practical limitation is on the actuator inputs. Actuator inputs can be limited because of their size and cost, particularly in large space structures. In the basic formulation of the optimal control problem, outlined in the previous two sections none of these constraints were included. In the definition of the performance index, both the actuator inputs and the time were assumed to be unbounded. However, by adjusting the scaling parameters in the weighting matrices θ^s, control objectives can be achieved with bounds on the actuator inputs and/or the settling time. The trade is between the actuator inputs and the settling time. Larger actuator inputs can reduce the settling time or, by increasing the settling time, the demands on the actuators can be reduced. To study the effects of changes in the θ^s on the control parameters, an equivalent damping parameter as defined by the logarithmic

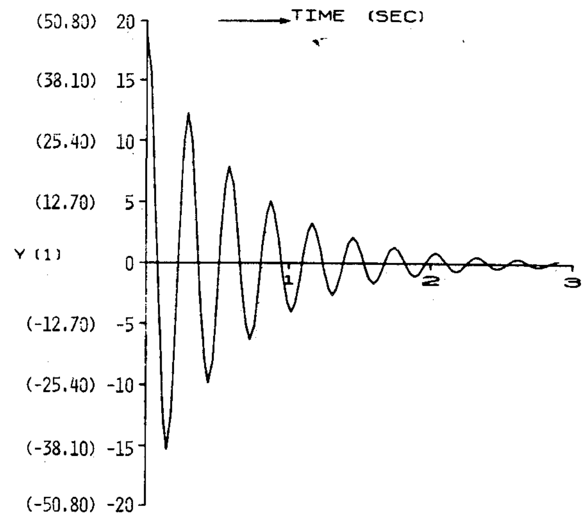

Fig. 2a Truss tip displacement, case 1, in. (cm).

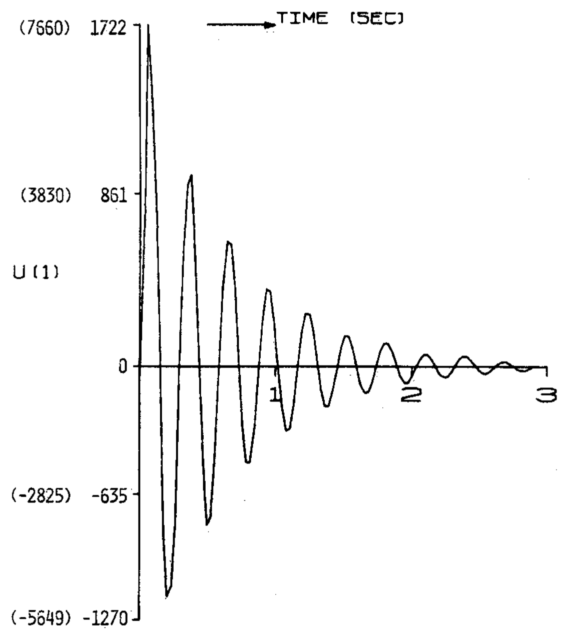

Fig. 2b Actuator input at the tip, case 1, lbf (N).

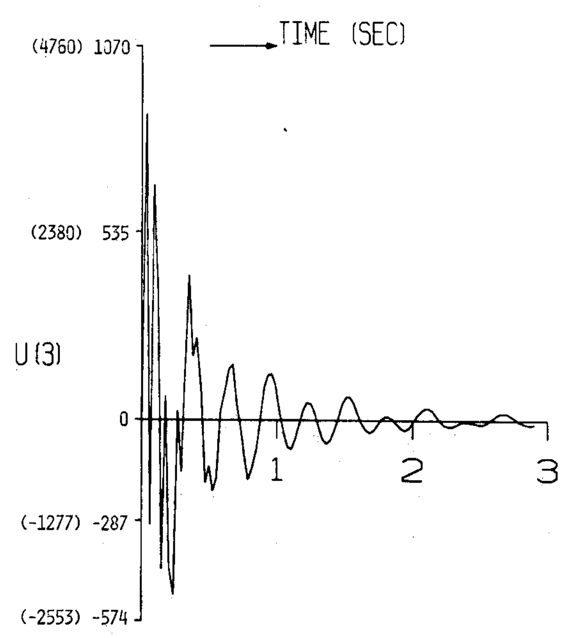

Fig. 2c Actuator input at the middle node, case 1, lbf (N).

decrement will be monitored

$$\bar{\zeta}=\frac{1}{2\pi}\ell n\left[\frac{y_P}{y_Q}\right] \tag{25}$$

where $\bar{\zeta}$ is the equivalent damping parameter, which includes the actual damping and the damping introduced by the control system. y_p and y_q are the amplitudes of the dynamic response in two consecutive cycles. For a given control system and structure, the variation of $\bar{\zeta}$ with the θ^s can be monitored.

To test the effects of the parametric changes of the weighting matrices and the structural variables, the truss shown in Fig. 1 was selected for vibration control. The total length of the truss is 254 cm (100 in.) and it is divided into two equal bays. It is a cantilever truss with depth 91.44 cm (36 in.) at the base and 60.96 cm (24 in.) at the tip. The truss is fixed at the base and free everywhere else. It is assumed to move only in its plane, giving 2 degrees of freedom per node. Excluding the 4 degrees of freedom at the supports (base), the truss has 8 degrees of freedom in the configuration space (displacements) and 16 degrees of freedom in the state space (displacements and velocities). A nonstructural mass of 0.585 kg (1.29 lb·s²/in.) per node was assigned to all the nodes except the two at the base. The truss is made of aluminum with $E=68.94\times10^9$ N/m² (10.0×10^6 psi) and $\rho=7.1635$ kg/m³ (0.1 lb/in.³) weight density.

The feedback control system on the structure consists of four actuators located at nodes 1-4. The sensor locations are assumed to coincide with the actuators. The actuator forces are to be applied only in the transverse direction of the truss, in which case $\boldsymbol{R}$ is a 4×4 matrix. Two primary truss designs are considered for this study. The first design, which will be called the original structure, is a nonoptimal structure with all the truss elements having the same cross-sectional area. The second one (called the optimal structure) is the same structure, but its weight is minimized with the fundamental frequency constraint. Table 1 gives the details of the member cross-sectional areas and the natural frequencies for the two structures.

Table 2 gives the concentrated nonstructural masses at the nodes for both the structures.

As can be seen from the two tables, the structural mass is quite insignificant compared to the nonstructural mass. The optimal and nonoptimal structures have the same fundamental frequency, but the optimal structure is about 30% lighter. Even though there is a significant reduction in weight due to the optimization, it is accompanied by a movement of the higher frequencies to the lower frequency. In some dynamic

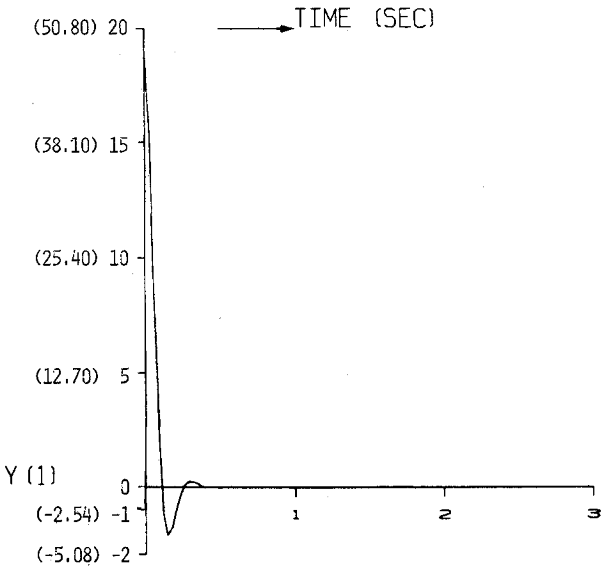

Fig. 3a Truss tip displacement, case 2, in. (cm).

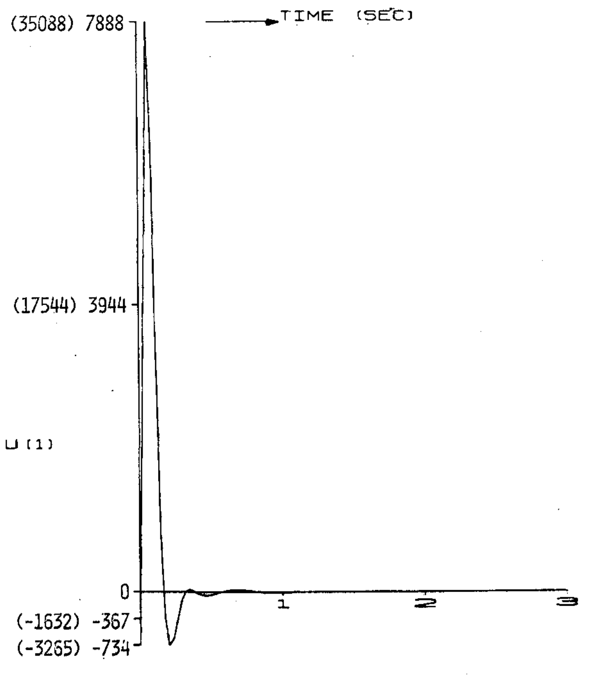

Fig. 3b Actuator input at the tip, case 2, lbf (N).

Table 2 Nonstructural mass

Node no.	Mass, kg (lb·s²/in.)
1	0.585 (1.29)
2	0.585 (1.29)
3	0.585 (1.29)
4	0.585 (1.29)

Table 3 Control response cases studied[a]

Case	Structure	$\frac{\theta_M}{\theta_{M0}}=\frac{\theta_K}{\theta_{K0}}$	$\frac{\theta_R}{\theta_{R0}}$
1	Orig	0.1	1.0
2	Orig	1.0	1.0
3	Opt	0.1	1.0
4	Opt	1.0	1.0

[a] $\theta_{M0}=\theta_{K0}=\theta_{R0}=1/\sqrt{2}$.

Table 4 Details of the initial state

Original, cm (in.)		Optimal, cm (in.)	
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
−6.71	(−2.64)	−5.92	(−2.33)
47.98	(18.89)	48.21	(18.98)
6.71	(2.64)	5.92	(2.33)
47.98	(18.89)	48.21	(18.98)
−6.38	(−2.51)	−3.66	(−1.44)
17.93	(7.06)	20.22	(7.96)
6.38	(2.51)	3.66	(1.44)
17.93	(7.06)	20.22	(7.96)

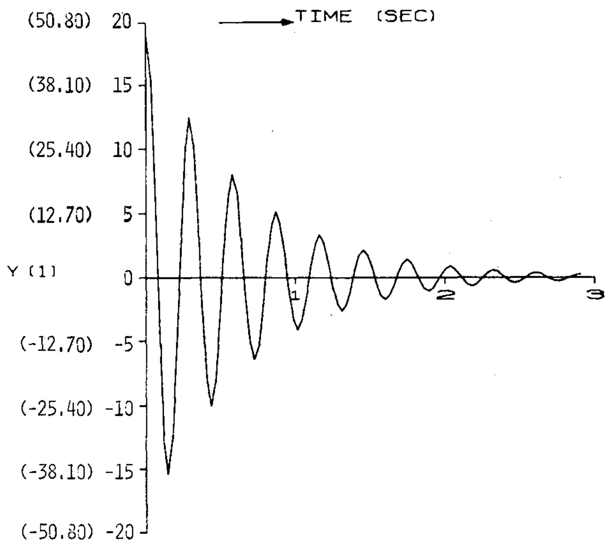

Fig. 4a Truss tip displacement, case 3, in. (cm).

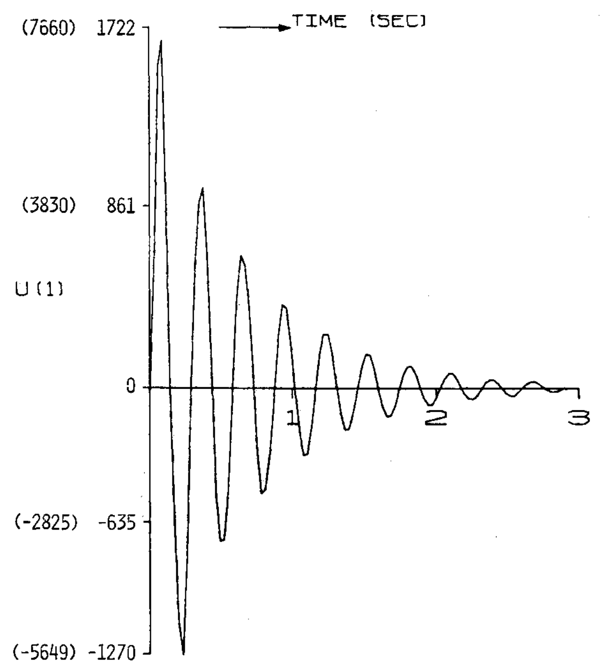

Fig. 4b Actuator input at the tip, case 3, lbf (N).

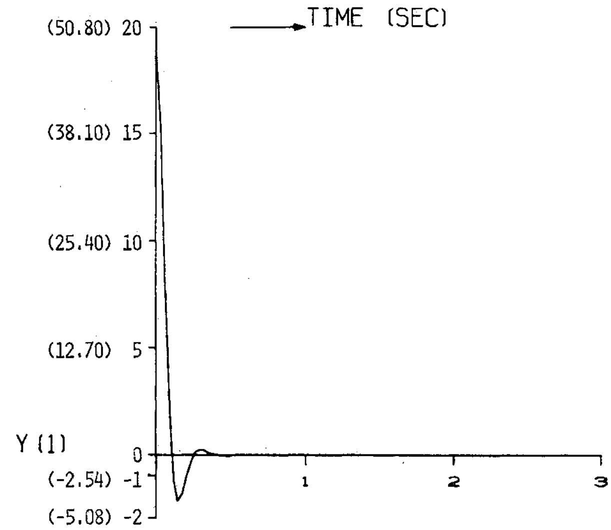

Fig. 5a Truss tip displacement, in case 4, in. (cm).

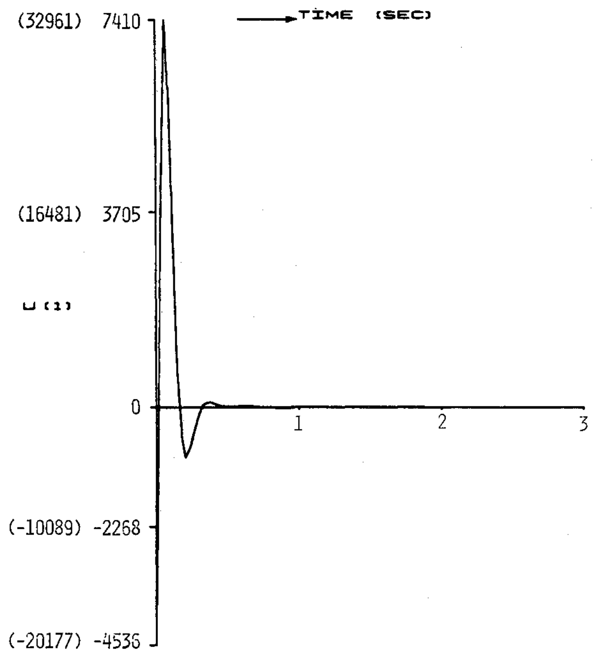

Fig. 5b Actuator input at the tip, case 4, lbf (N).

response problems, this may not be a desirable feature. In such cases, multiple frequency constraints have to be imposed. For this particular study, the closing in of the frequencies is not significant enough to present difficulties.

A list of the cases studied in this paper is given in Table 3. These cases include both the original and the optimal structure. In all cases, the initial state was assumed to be a static displacement vector resulting from the application of a 44,482.2N (10,000 lb)force at each of the actuator locations and a zero velocity vector. The details of the initial state for both the structures are given in Table 4.

In all cases studied, the system is controllable and observable. The full system formulation (no modal reduction) was used in all cases. The open- and closed-loop eigenvalues are given (for cases 1 and 3 as examples) in Table 5.

The equivalent damping [as calculated from Eq. (25)] resulting from the use of feedback controls is given in Table 6 for cases 1 and 2. In these cases, only the multipliers of the $\boldsymbol{Q}$ matrix were assumed to vary while keeping the $\boldsymbol{R}$ matrix multipliers constant. As the value of the $\boldsymbol{Q}$ multipliers increases, the equivalent damping increases. As the equivalent damping increases, the settling time decreases. However, the power requirements of the actuators (controllers) increase. The $\boldsymbol{\theta}_R$ matrix is a diagonal matrix and it is an identity matrix when there are no bounds on the input of the actuators. If some of the actuators have amplitude constraints (individual actuators) on their inputs, then the corresponding elements on the diagonal would be different from unity, but nevertheless positive multipliers. A comprehensive description outlining the procedure for selecting these scalar multipliers will be included in a future study.

The tip displacement in the transverse direction is plotted against time for all four cases (see Figs. 2a-5a). The actuator input at the tip is plotted against time for all cases (see Figs. 2b-5b). Due to symmetry, actuators 1 and 2 have the same input. Similarly, the input of actuators 3 and 4 is the same. In the interest of saving space, a plot of the third actuator input is given only for case 1 (see Fig. 2c).

The performance index PI is defined by

$$\mathrm{PI} = X_0^T \boldsymbol{P} X_0$$

Table 5 Open- and closed-loop eigenvalues (for cases 1 and 3)

Original structure			Optimized structure		
Open loop	Closed loop		Open loop	Closed loop	
Imag[a]	Real	Imag	Imag[a]	Real	Imag
275.0	−12.8	274.0	222.0	−0.794	222.0
−275.0	−12.8	−274.0	−222.0	−0.794	−222.0
247.0	−13.3	247.0	208.0	−0.318	208.0
−247.0	−13.3	−247.0	−208.0	−0.318	−208.0
238.0	−14.8	239.0	125.0	−1.40	125.0
−238.0	−14.8	−239.0	−125.0	−1.40	−125.0
227.0	−0.398	227.0	111.0	−5.18	111.0
−227.0	−0.398	−227.0	−111.0	−5.18	−111.0
167.0	−1.87	167.0	83.5	−5.80	83.4
−167.0	−1.87	−167.0	−83.5	−5.80	−83.4
94.7	−1.62	94.7	76.7	−4.09	76.7
−94.7	−1.62	−94.7	−76.7	−4.09	−76.7
78.4	−5.47	78.4	58.8	−4.10	58.8
−78.4	−5.47	−78.4	−58.8	−4.10	−58.8
21.6	−1.53	21.6	21.6	−1.53	21.6
−21.6	−1.53	−21.6	−21.6	−1.53	−21.6

[a]Real part equals zero.

Table 6 Equivalent damping[a]

Case no.	$\theta_M/\theta_{M0} = \theta_K/\theta_{K0}$	$\bar{\zeta}_{EQ}$
1	0.1	0.070
2	1.0	0.687

[a]$\theta_R/\theta_{R0} = 1$.

Table 7 Performance index

Case no.	PI, J	(in.-lb)
1	189.3	(1,675.1)
2	2549.3	(22,562.8)
3	195.5	(1,730.7)
4	2633.1	(23,304.8)

Table 7 gives the value of the performance index for all the cases.

Summary and Conclusions

A brief outline of the optimal control and structural design problem is presented from a structural dynamicist's point of view. Selection of the weighting matrices in the definition of the performance index and their implication in control response is discussed in detail. By introducing simple scaling parameters, the weighting matrices were used effectively to achieve the desired control objectives. A number of case studies were made using a simple truss structure. A nonoptimal and an optimal truss were used in this study with the same set of actuators. Modification of the structural parameters did not significantly alter the control design in this study. However, in the presence of external disturbances (persistent) in addition to the initial conditions, the structural parameter changes are expected to have a more pronounced effect on the control systems design. This aspect of combined structural/control optimization will be the subject of our future study.

References

[1]*Modeling, Analysis and Optimization Issues for Large Space Structures,* Proceedings of NASA/AFOSR Workshop, edited by L. D. Pinson, A. K. Amos, and V. B. Venkayya, Williamsburg, VA, May 1982.

[2]*Dynamics and Control of Large Structures*, 4th VPI&SU/AIAA Symposium, Blacksburg, VA, June 1983, edited by L. Meirovitch.

[3]D'Azzo, J. J. and Houpis, C. H., *Linear Control System Analysis and Design, Conventional and Modern*, 2nd ed., McGraw-Hill Book Co., New York, 1981.

[4]Athans, M. and Falb, P. L., *Optimal Control—An Introduction to the Theory and Its Applications*, McGraw-Hill Book Co., New York, 1966.

[5]Sandell, N. R. and Athans, M., "Modern Control Theory—A Self Study Subject-Computer Manual," Center for Advanced Study, Massachusetts Institute of Technology, Cambridge, MA.

[6]Meirovitch, L., *Computational Methods in Structural Dynamics*, Sijthoff and Noordhoff, Rockville, MD, 1980.

[7]Calico, R.A. and Miller, W. T., "Decentralized Control for a Flexible Spacecraft," AIAA Paper 82-1404, Aug. 1982.

[8]Miller, D. F. and Wells, W. R., "Modal Control of Structural Systems," AFWAL-TR-84-3054.

[9]Venkayya, V. B. and Tischler, V. A., "Optimization of Structures with Frequency Constraints," *Proceedings, Computer Methods for Nonlinear Solids and Structural Mechanics*, edited by N. S. Atluri, ASME, New York, June 1983.

[10]Khot, N. S., "Optimization of Structures with Multiple Frequency Constraints," *International Journal of Computers and Structures*, to be published.

[11]Kamat, M. P., Venkayya, V. B., and Khot, N. S., "Optimization with Frequency Constraints—Limitations," *Journal of Sound and Vibration*, Vol. 91, No. 1, 1983, pp. 147-154.